MID-SEMESTER EXAMINATION
B. MATH III YEAR/M. MATH II YEAR
II SEMESTER, 2006-2007
STOCHASTIC PROCESSES

[Questions 1,2,5 and 6 carry 15 marks each. Questions 3 and 4 carry 20 marks each]

1. Assuming Stirling's formula ($\lim_{n\to\infty} \frac{n!}{e^{-n}n^{n+1/2}} = \sqrt{2\pi}$) and the limit theorem for Markov chains show that a symmetric random walk on the set of all integers is null recurrent.

2. Are the following statements true? If so, give a proof. If not, give a counter-example.

a) i recurrent, $(i \longleftrightarrow j) \Rightarrow j$ recurrent b) i transient, $(i \rightarrow j) \Rightarrow j$ transient

c) $p_{ij}^{(n)} > 0 \Rightarrow f_{ij}^{(n)} > 0$ d) $f_{ij}^{(n)} > 0 \Rightarrow p_{ij}^{(n)} > 0$

3. Prove that $\gcd\{n : f_{ii}^{(n)} > 0\} = \gcd\{n : p_{ii}^{(n)} > 0\}$

4. Find all stationary distributions of a Markov chain with transition matrix

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 1/3 & 2/3 & 0 & 0 \\ 0 & 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & 0.9 \\ 0 & 0 & 0 & 0.8 & 0.2 \end{bmatrix}$$

5. Classify the states of a Markov chain with transition matrix

$$\begin{bmatrix} 1/2 & 0 & 1/8 & 1/4 & 1/8 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 0 & 0 & 1/3 & 2/3 \end{bmatrix}$$

6. Let $\{X_n\}$ be a Markov chain with state space $S = \{0, 1, 2, ...\}$. Suppose $p_{i,i+1} + p_{i,0} = 1$ for all $i \geq 0$ and $p_{i,i+1} (= p$ say, with $0 < p < 1)$ is independent of i. Classify the states of this Markov chain and find $f_{0,0}^{(n)}$ for each n. What is the mean return time to state 0?